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AN UNWANTED POSSIBILITY

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Abstract: In this paper I consider three theories of modality and ask whether they can account for the impossibility of the round square. I argue that none of them can, which leaves us with the uncomfortable realisation that the impossibility of one of our classical examples of impossible objects is not accounted for by our best theories of modality.

Keywords: Conceivability, Epistemic Modality, Essential Properties, Impossible Objects, Round Square, Theories of modality.

UNA POSIBILIDAD INDESEADA

Resumen: En este artículo discuto tres teorías de la modalidad, planteando si pueden ofrecer una explicación satisfactoria de la imposibilidad del círculo cuadrado. Sostengo que ninguna de ellas es capaz de cumplir dicha tarea, lo que nos deja con la incómoda conclusión de que nuestras mejores teorías de la modalidad no dan cuenta de la imposibilidad de uno de los ejemplos clásicos de objeto imposible.

Palabras clave: Círculo cuadrado, concebibilidad, modalidad epistémica, objetos imposibles, propiedades esenciales, teorías de la modalidad.

... is no less absurd than believing that a circle
has taken on the nature of a square.
(Spinoza)

How do we, in general, reach a conclusion? We assume some premisses and from there, with the help of a theory that we subscribe to, we reach said conclusion. The conclusion I discuss in this paper is that the existence of a round square is impossible. To do so, I consider three modal theories and ask whether they lead to this conclusion from commonly held premisses.

The first theory of modality I consider in this paper links possibility with conceivability. One might hold that we cannot conceive of something which is both round and square. Hence, the round square is inconceivable and, so the argument goes, its existence therefore impossible. The argument assumes that inconceivability implies impossibility and that hence, by classical logic, possibility implies conceivability. Then all the features about this world (which are actual and hence possible) are conceivable. This demands some ideal agent for the understanding of `conceivable'. However, the argument `the round square is inconceivable and hence impossible' lacks a reason why an ideal agent cannot conceive of a round square. Therefore, even if the `inconceivability implies impossibility' assumption can be upheld, the above argument is not strong enough to feature as a reason for the impossibility of the existence of the round square.

The second theory of modality discussed in this paper is the theory of epistemic modality. It might be claimed that the non-existence of a round square is an epistemic necessity. P is an epistemic necessity if an ideal agent without any information about the world arrives, by reasoning alone, at P. That is, P is an epistemic necessity if P can be known a priori. By the classical possibility-necessity dualism,[1] it follows that P is an epistemic impossibility if non-P is an epistemic necessity, i.e. non-P is known a priori. In this light, the round square problem reduces to the question of whether the non-existence of the round square is knowable a priori.

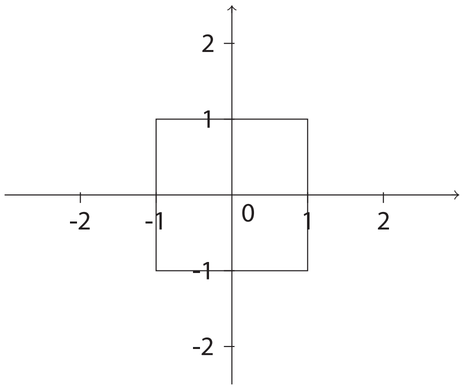
A priori justifications are fallible. For a long time it was held that Euclidean geometry is a priori true. This was refuted by the emergence of non-Euclidean geometries. I shall do something similar. I give a construction of an extended metric space[2] in which a round square exists. This construction serves as an argument that the non-existence of a round square is not knowable a priori.

1 The dualism principle holds, that P is possible if and only if it is not necessary that non-P, i.e. $\Diamond P \leftrightarrow \neg\Box\neg P$.

2 *A metric space* is a mathematical concept and consists of a set of points and an associated metric. It is relevant for my construction to use certain definitions as well, which extend the standard conception of a metric space.

A square is a geometrical figure with four sides of equal length and two diagonals of equal length. The curvature of a line is given by the inverse of its radius, and a line is said to be round if it has constant curvature. A radius is the distance of every point on a line segment to a point. Distances are given by metrics, which are functions that, given any two points, generate a non-negative real number. Note that all these definitions are mathematically perfectly acceptable.

Consider the two-dimensional plane $\mathbb{R}^2$. Define on it the maximums metric $D(x,y)=max\{\ |x_1\text{-}y_1|,\ |x_2 - y_2|\}$, and denote the metric space ($\mathbb{R}^2$, *D*) plus its extension by the above definitions by *M*. In *M*, consider the set *S* of all points with distance 1 to the origin. This set is classically called the unit-circle. Plotting *S* in *M* looks like this:



In *M*, *S* has four sides of length 2, and two diagonals of length 2, i.e. *S* is a square in *M*. Furthermore, every point in *S* has distance 1 to the origin, i.e. *S* has constant curvature =1. Hence, in *M*, *S* is round. Therefore, in *M*, S is round and a square, i.e. *S* is a round square in *M*. This concludes the construction and ends my argument that the non-existence of a round square is not known a priori.

Lastly I consider the theory which approaches modality by means of essential properties. It might be argued, that *M* erases what it means to be a square, contrasted to what it means to be round. According to this argument, it is an essential property of the concept of square (or roundness) that it excludes roundness (or squareness), and, so the argument goes, the round square is therefore conceptually impossible. This train of thought runs the risk of circularity since the classical understanding of `essential property' relies on modal notions:

P is essential for *x* if and only if *x* necessarily has *P*.

Because a proponent of the conceptual argument would have to argue for the right hand side of the equivalence he/she may neither claim the left hand side without further argument, nor may he/she argue for the left hand side by presupposing the right hand side. To avoid these difficulties, he/she might try to embrace a non-classical understanding of `essential property', such as the definitorial or the explanatory characterisation. However, general criticisms, such as the seeming subjective turn of the concept `essential property' of these characterisations, and the concrete counter-evidence that `square' has both been defined and been explained above without referring to its `non-roundness' property, have yet to be answered.

To conclude, given the commonly held premisses and the relevant theories of modality discussed in this paper, the conclusion that the round square is an impossible object does not follow. The approach via conceivability lacks the needed argument that an ideal agent cannot conceive of a round square. The epistemic treatment of modality is based on the a priori knowability of the statement, which is, for the round square and in the light of *M*, not given. The conceptual argument lacks acceptable reasons why non-roundness (non-squareness) is an essential property of a square (something round).

This conclusion forces us to do one of three things. To save our preferred modal theory while simultaneously trying to uphold the impossibility of the existence of a round square we could change the premisses. For example in an account of modality via conceivability we could claim that we assumed *as a premiss* that the standard metric is used. Such moves are ad hoc. Instead we could accept the force of this paper and try to improve our theories of modality in such a way that they can account for the non-existence of a round square even under the given premisses. Lastly we could accept that the round square is not impossible after all, that I have in fact constructed a round square. This would force us to hold that *the* classic example of an impossible object is, in fact, not an example at all.[3]

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